Received: *November 3, 1999*, Accepted: *January 14, 2000*

Tachyonic modes on type-0 NS5-branes*

Yosuke Imamura

Center for Theoretical Physics, Massachusetts Institute of Technology
Cambridge, MA 02139-4307, USA, and
Yukawa Institute for Theoretical Physics
Kyoto University, Kyoto 606-8502, Japan
E-mail: imamura@ctp.mit.edu

Abstract: Via T-duality, a stack of unwrapped type 0 NS5-branes is transformed into a Kaluza-Klein monopole with A_n type singularity at its center. The spectrum of twisted modes at the singularity contains tachyonic modes. We show that, in certain parameter region, this tachyonic spectrum is completely reproduced as modes of the bulk tachyon field localized on a classical NS5-brane solution. In passing, we show how twisted modes at the singularity reproduce gauge fields on stacks of NS5-branes.

Keywords: Superstrings and Heterotic Strings, p-branes, String Duality.

*This work was supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DE-FC02-94ER40818 and by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture (#9110).

Contents

1. Introduction

NS5-branes are non-perturbative objects in string theory and, unlike the string itself and D-branes, we cannot directly analyze them by worldsheet CFT. They attract great interest in connection with the mysterious interacting quantum theory without gravity that they carry on their world volume [1]–[5].

The massless spectra on type-IIA and type-IIB NS5-branes were first obtained in [6]. In that work, it is shown that on an NS5-brane classical solution there are zero modes of RR fields representing a vector field (type IIB) or ones representing a self-dual two-form field and a scalar field (type IIA). These are not whole spectra on NS5-branes. By an S-duality argument, we know that $\mathrm{U}(Q)$ gauge symmetry is realized on type-IIB NS5-branes. Similarly, in type-IIA case, it is expected that the gauge symmetry associated with the tensor multiplet is enhanced into a 'non-abelian' gauge symmetry. For the present, however, we do not know how such a theory is described.

Type-0A and type-0B theories also contain NS5-branes. By applying the method of [6], we obtain the following massless spectra. On a type-0B NS5-brane there are 4 scalar fields representing fluctuations and two $\mathrm{U}(1)$ gauge fields. On a type-0A NS5-brane there are four fluctuation modes, one unconstrained two-form field and two scalar fields. If Q type-0 NS5-branes coincide, in imitation of type-II NS5-branes, we can guess the gauge symmetries to be enhanced into $\mathrm{U}(Q) \times \mathrm{U}(Q)$. For type-0A NS5-branes, of cause, this statement has just a superficial sense.

There is another way to obtain massless spectra on NS5-branes. Via T-duality, unwrapped NS5-branes are transformed into Kaluza-Klein monopoles [7]. So, we can obtain massless spectra on NS5-branes as massless modes on Kaluza-Klein monopoles. Recently, this method is used to obtain massless spectra on separated type-0 NS5-branes [8, 9]. In general, Kaluza-Klein monopoles have A_n type singularities and, in addition to zero modes of bulk fields, we should take account of twisted modes of strings at the singularities. In section 4, we show that all 'Cartan part' of gauge fields on Q coincident NS5-branes are reproduced in this way. (For type-IIA and type-0A NS5-branes, we define the 'Cartan part' as the gauge symmetry which remains when all branes separate from each other.)

As we demonstrate later, the analysis of the twisted modes shows that tachyonic fields also exist on type-0 coincident NS5-branes. The purpose of this paper is to find the counterpart of these modes in the supergravity description of the NS5-branes. If it is possible, they must be modes of the bulk tachyon field since massless and massive fields cannot generate tachyonic modes. In section 5, we analyze modes of the bulk tachyon field on a NS5-brane classical solution background and show that the tachyonic spectrum of twisted modes is completely reproduced, at least in a certain parameter region.

Of cause, the existence of tachyonic modes means that the configuration is unstable. It is interesting to ask what happens if the tachyons condensate. At this time, however, we have no answer to this question and in this paper we concentrate our attention to an analysis on the unstable vacuum without tachyon expectation values. We would like to stress that this unstability may not exist in the case of NS5-branes on non-compactified spacetime. Because we will use the T-duality, the compactification of one transverse direction is crucial for our analysis. In fact, we obtain different tachyonic spectra depending on the monodromy around S^1. This fact suggest that wrapped D-strings with both ends on the NS5-branes contribute the spectra.

2. T-duality of NS5-branes

In this section, we briefly review the T-duality between unwrapped NS5-branes and Kaluza-Klein monopoles with keeping our eyes on geometry and symmetry. First, we discuss familiar T-duality, namely, 0A/0B and IIA/IIB duality of unwrapped NS5-branes in S^1 compactified string theory. We will mention the 'exotic' IIA/0B and IIB/0A duality suggested in [10] at the end of this section.

The following arguments applicable to type-IIA, -IIB, -0A and -0B theories. Let us assume NS5-branes extending along $x^0, \ldots, x^5$ and the x^9 direction compactified on S^1 with radius R. These NS5-branes are magnetically coupled to NS-NS 2-form field B_{i9} $(i = 6, 7, 8)$. Because B_{i9} is transformed into the metric component g_{i9} by T-duality, the dual to a stack of Q NS5-branes is a Kaluza-Klein monopole with charge Q [7].

The metric of generic parallel Kaluza-Klein monopoles is given by [11]

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + v(x^i)(dx^i)^2 + v^{-1}(x^i)\left(dx^9 + A_i(x^i)dx^i\right)^2, \tag{2.1}$$

where x^μ ($\mu = 0, \ldots, 5$) and x^i ($i = 6, 7, 8$) are coordinates of the parallel directions and the transverse directions, respectively. The coordinate x^9 parameterizes the compactified direction and takes value in $0 \le x^9 < 2\pi\widetilde{R}$, where $\widetilde{R}$ is compactification radius. If we denote the monopole density by $\rho_{\rm mon}(x^i)$, a magnetic potential $v(x^i)$ and an electric potential $A_i(x^i) \sim g_{9i}$ are obtained by solving the following differential equations:

$$\sum_{i=6}^{8}\left(\frac{\partial}{\partial x^i}\right)^2 v(x^i) = -2\pi\widetilde{R}\rho_{\rm mon}(x^i), \qquad \epsilon_{ijk}\partial_j A_k(x^i) = \partial_i v(x^i). \tag{2.2}$$

When the charge Q concentrates at $x^i = 0$, by setting $\rho_{\rm mon}(x^i) = Q\delta^3(x^i)$, we obtain

$$v(x^i) = 1 + \frac{\widetilde{R}Q}{2|x^i|}. \tag{2.3}$$

In a part of this manifold where $|x^i|$ is much smaller than $\widetilde{R}Q$, we can neglect the first term on the left hand side of (2.3). If we introduce a radial coordinate $r = \sqrt{2\widetilde{R}Q|x^i|}$ representing the geodesic distance from the point $x^i = 0$, and the line element $d\Omega_2$ on unit S^2, (2.1) is rewritten as

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + \frac{r^2}{4}\left[d\Omega_2^2 + \frac{4}{\widetilde{R}^2}\left(\frac{dx^9}{Q} + a\cdot d\Omega_2\right)^2\right], \tag{2.4}$$

where a_i is the unit charge magnetic monopole gauge configuration on the unit S^2 and A_i is written as $A_i = (Q/|x^i|)a_i$. The metric (2.4) represents an orbifold $\mathbb{R}^4/\mathbb{Z}_Q$ since if the period of x^9 were $0 \le x^9 < 2\pi\widetilde{R}Q$, which is Q times what it actually is, it would represent flat $\mathbb{R}^4$ [12]. By moving to a cartesian coordinate, we can show that the $\mathbb{Z}_Q$ is subgroup of one of two SU(2) factors of SO(4) symmetry of the divided $\mathbb{R}^4$. We will refer to the SU(2) containing $\mathbb{Z}_Q$ as $\mathrm{SU}(2)_L$ and to the other as $\mathrm{SU}(2)_R$. If $Q \ge 3$, $\mathrm{SU}(2)_L$ is broken to U(1) by the $\mathbb{Z}_Q$ orbifolding. Let $\mathrm{U}(1)_L$ denote this symmetry. Even if $Q \le 2$, $\mathrm{SU}(2)_L$ is broken to $\mathrm{U}(1)_L$ on the whole manifold described by the metric (2.1). This $\mathrm{U}(1)_L$ symmetry represents a shift of the x^9 coordinate. On the other hand, $\mathrm{SU}(2)_R$ represents rotation of the base $\mathbb{R}^3$ parameterized by x^i. Therefore, the isometry of the Kaluza-Klein monopole solution is

$$P(1,5) \times \mathrm{SU}(2)_R \times \mathrm{U}(1), \tag{2.5}$$

where $P(1,5)$ represents the Poincaré symmetry on a 1 + 5-dimentional Minkowski space.

The symmetry of NS5-branes in uncompactified ten-dimensional spacetime is $P(1,5) \times \mathrm{SO}(4)$. Let $\mathrm{SU}(2)_1$ and $\mathrm{SU}(2)_2$ denote the two SU(2) factors of this SO(4).

Due to compactification of one transverse direction, SO(4) is broken to SO(3) $\sim$ SU(2). We call it $\mathrm{SU}(2)_D$ since it is the diagonal subgroup of $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$. Therefore, the symmetry of a NS5-brane configuration in S^1 compactified spacetime is

$$P(1,5) \times \mathrm{SU}(2)_D\,. \tag{2.6}$$

In the low-lying supergravity level, it is known that S^1 compactified type-IIA theory and type-IIB theory have the same effective action [13]. Because the classical Kaluza-Klein monopole solution (2.1) does not depend on x^9, it is also a solution of nine dimensional supergravity. Therefore, by the T-duality of supergravity, we can obtain a solution in nine-dimensional supergravity coupled to B_{i9} magnetically. This solution is lifted up to ten-dimensional solution independent from x^9. It is called a 'smeared' NS5-brane solution. The symmetry of this solution is

$$P(1,5) \times \mathrm{SU}(2)_D \times \mathrm{U}(1)_S\,, \tag{2.7}$$

where $\mathrm{U}(1)_S$ represent a shift along x^9 direction.

Let us compare the symmetry of the NS5-brane configuration and the Kaluza-Klein monopole configuration. The symmetry $P(1,5) \times \mathrm{SU}(2)_D$ of the NS5-branes is identified with the symmetry $P(1,5) \times \mathrm{SU}(2)_R$ on the Kaluza-Klein monopole side. Because $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_S$ factors in (2.5) and (2.7) are associated with the gauge field g_{9i}, they are transformed by T-duality into non-geometric symmetries associated with B_{i9}.

Finally, we will mention the exotic T-dualities. To relate type-0 theory and type-II theory by the T-duality, we should introduce non-trivial monodromies around S^1. Namely, we need monodromy $(-1)^{F_R}$ for type-0 theory and $(-1)^{F_S}$ for type-II theory, where the operators F_R and F_S are the right moving worldsheet fermion number and the spacetime fermion number, respectively [10]. In what follows, we call a circle with monodromy $(-1)^{F_S}$ as $S^1_{(F_S)}$ and one with monodromy $(-1)^{F_R}$ as $S^1_{(F_R)}$. Via this duality, a stack of Q unwrapped NS5-branes is transformed into a Kaluza-Klein monopole with charge $2Q$, rather than Q [9]. The central part of the Kaluza-Klein monopole is described as an orbifold $\mathbb{R}^4/\mathbb{Z}_{2Q}$. Because of the non-trivial monodromy of the S^1 cycle, the dividing $\mathbb{Z}_{2Q}$ is generated by $(-1)^{F_S}\gamma$ (type II) or $(-1)^{F_R}\gamma$ (type-0), where γ is a generator of $\mathbb{Z}_{2Q} \subset \mathrm{SU}(2)_L$. In what follows, we call them $\mathbb{Z}_{2Q}^{(F_S)}$ (generated by $(-1)^{F_S}\gamma$) and $\mathbb{Z}_{2Q}^{(F_R)}$ (generated by $(-1)^{F_R}\gamma$). These duality relations between NS5-branes and Kaluza-Klein monopoles are summarized in table 1.

3. Tachyonic twisted modes

In this section, we give a tachyonic part of the spectrum of twisted modes of orbifold $\mathbb{R}^4/\Gamma$, where Γ is one of $\mathbb{Z}_N$, $\mathbb{Z}_N^{(F_S)}$ and $\mathbb{Z}_N^{(F_R)}$. The integer N is equal to Q or $2Q$ depending on the type of T-duality. We need to consider only the NS-NS sector since other sectors does not contain tachyonic modes.

JHEP01(2000)018

Q unwrapped NS5-branes	Kaluza-Klein monopoles
type IIA on S^1	type IIB on $\mathbb{R}^4/\mathbb{Z}_Q$
type IIB on S^1	type IIA on $\mathbb{R}^4/\mathbb{Z}_Q$
type IIA on $S^1_{(F_S)}$	type 0B on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_R)}$
type IIB on $S^1_{(F_S)}$	type 0A on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_R)}$
type 0A on S^1	type 0B on $\mathbb{R}^4/\mathbb{Z}_Q$
type 0B on S^1	type 0A on $\mathbb{R}^4/\mathbb{Z}_Q$
type 0A on $S^1_{(F_R)}$	type IIB on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_S)}$
type 0B on $S^1_{(F_R)}$	type IIA on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_S)}$

Table 1: T-duality between unwrapped NS5-branes and Kaluza-Klein monopoles.

Let us introduce complex coordinates z^A $(A = 1, 2)$ on the divided $\mathbb{R}^4$, which is rotated by $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ as

$$U \to g_L U g_R\,, \qquad U \equiv \begin{pmatrix} z^1 & z^2 \\ -\overline{z}_2 & \overline{z}_1 \end{pmatrix}, \qquad g_L \in \mathrm{SU}(2)_L\,, \qquad g_R \in \mathrm{SU}(2)_R\,. \tag{3.1}$$

A generator γ of discrete group Γ acts on the complex coordinates as follows:

$$\gamma:\ z^A \to \exp\left(\frac{2\pi i}{N}\right) z^A\,, \qquad \overline{z}_A \to \exp\left(-\frac{2\pi i}{N}\right)\overline{z}_A\,. \tag{3.2}$$

At the fixed point of this orbifold, $N-1$ twisted modes appear. We label them as $k = 1, \ldots, N-1$. For each k, z^A and $\overline{z}_A$ satisfy the boundary conditions

$$z^A(\sigma + 2\pi) = e^{2\pi i k/N} z^A(\sigma)\,, \qquad \overline{z}_A(\sigma+2\pi) = e^{-2\pi i k/N}\overline{z}_A(\sigma)\,, \tag{3.3}$$

and are expanded as

$$z^A = l_s \sum_{m=-\infty}^{\infty}\left(\frac{ie^{i(m-k/N)(\tau-\sigma)}}{m-k/N} z^A_{m-k/N} + \frac{ie^{i(m+k/N)(\tau+\sigma)}}{m+k/N}\tilde{z}^A_{m+k/N}\right), \tag{3.4}$$

$$\overline{z}_A = l_s \sum_{m=-\infty}^{\infty}\left(\frac{ie^{i(m+k/N)(\tau-\sigma)}}{m+k/N}\overline{z}_{A,m+k/N} + \frac{ie^{i(m-k/N)(\tau+\sigma)}}{m-k/N}\overline{\tilde{z}}_{A,m-k/N}\right). \tag{3.5}$$

In a same way, boundary conditions and expansions of NSR fermions ψ^A and $\overline{\psi}_A$, which are superpartners of z^A and $\overline{z}_A$, are given by

$$\psi^A(\sigma+2\pi) = -e^{2\pi i k/N}\psi^A(\sigma)\,, \qquad \overline{\psi}_A(\sigma+2\pi) = -e^{-2\pi i k/N}\overline{\psi}_A(\sigma)\,, \tag{3.6}$$

and

$$\psi^A = \sum_{m=-\infty}^{\infty}\left(\psi^A_{m+\frac{1}{2}-\frac{k}{N}} e^{i(m+\frac{1}{2}-\frac{k}{N})(\tau-\sigma)} + \tilde{\psi}^A_{m+\frac{1}{2}+\frac{k}{N}} e^{i(m+\frac{1}{2}+\frac{k}{N})(\tau+\sigma)}\right),$$

$$\overline{\psi}_A = \sum_{m=-\infty}^{\infty}\left(\overline{\psi}_{A,m+\frac{1}{2}+\frac{k}{N}} e^{i(m+\frac{1}{2}+\frac{k}{N})(\tau-\sigma)} + \overline{\tilde{\psi}}_{A,m+\frac{1}{2}-\frac{k}{N}} e^{i(m+\frac{1}{2}-\frac{k}{N})(\tau+\sigma)}\right). \tag{3.7}$$

First, we should give the energy and the $\mathrm{U}(1)_L$ charge of ground state. $\mathrm{U}(1)_L$ charge is necessary when we carry out the Γ projection. For this purpose, we can use the following ζ-function regularization formulae:

$$\sum_{m+a>0}(m+a)^0 = \left[\left[\frac{1}{2}-a\right]\right], \qquad \sum_{m+a>0}(m+a)^1 = -\frac{1}{2}\left[\left[\frac{1}{2}-a\right]\right]^2 + \frac{1}{24}, \tag{3.8}$$

where $[[x]]$ denotes the element of $\mathbb{Z}+x$ with the smallest absolute value. The choice between $\pm 1/2$ for $[[1/2]]$ does not affect the arguments below. Using this formula, we obtain the vacuum energy of each twisted sector as follows:

$$L_0 =: L_0 : -\frac{1}{2} + \left|\left[\left[\frac{k}{N}\right]\right]\right|. \tag{3.9}$$

The left moving part of $\mathrm{U}(1)_L$ charge $Q_{\mathrm{U}(1)_L}$ is defined as one satisfying the following relations:

$$\begin{aligned} [Q_{\mathrm{U}(1)_L}, \psi^A] &= \psi^A , & [Q_{\mathrm{U}(1)_L}, \overline{\psi}_A] &= -\overline{\psi}_A , \\ [Q_{\mathrm{U}(1)_L}, z^A] &= z^A , & [Q_{\mathrm{U}(1)_L}, \overline{z}_A] &= -\overline{z}_A . \end{aligned} \tag{3.10}$$

Such an operator is given by

$$Q_{\mathrm{U}(1)_L} = \sum_{m=-\infty}^{\infty}\sum_{A=1,2}\left(\frac{1}{-m+k/N} z^A_{m-\frac{k}{N}}\overline{z}_{A,-m+\frac{k}{N}} + \psi^A_{m+\frac{1}{2}-\frac{k}{N}}\overline{\psi}_{A,-m-\frac{1}{2}+\frac{k}{N}}\right). \tag{3.11}$$

Regularizing this by (3.8), we obtain

$$\begin{aligned} Q_{\mathrm{U}(1)_L} &= : Q_{\mathrm{U}(1)_L} : + 2\left[\left[\frac{k}{N}\right]\right] + 2\left[\left[\frac{1}{2}-\frac{k}{N}\right]\right] \\ &= \begin{cases} : Q_{\mathrm{U}(1)_L} : +1 , & [[k/N]] > 0 , \\ : Q_{\mathrm{U}(1)_L} : -1 , & [[k/N]] < 0 . \end{cases} \end{aligned} \tag{3.12}$$

Namely, the ground state has $Q_{\mathrm{U}(1)_L} = \pm 1$ depending on the signature of $[[k/N]]$. For the right moving part, we can get the following result by replacing k in the result of the left moving part by $-k$:

$$\widetilde{Q}_{\mathrm{U}(1)_L} = \begin{cases} : \widetilde{Q}_{\mathrm{U}(1)_L} : -1 , & [[k/N]] > 0 , \\ : \widetilde{Q}_{\mathrm{U}(1)_L} : +1 , & [[k/N]] < 0 . \end{cases} \tag{3.13}$$

Therefore, total $\mathrm{U}(1)_L$ charge $Q_{\mathrm{U}(1)_L} + \widetilde{Q}_{\mathrm{U}(1)_L}$ of the ground state is always zero.

Now, we should construct the Fock space by exciting the ground state with oscillators and should impose GSO and Γ projections. The result for each case is as follows. All results holds for A or B type theories.

Type II on $\mathbb{R}^4/\mathbb{Z}_Q$. The ground state of type-II theories is GSO odd and it should be excited by at least one fermionic oscillator. If we restrict our attention to the case with $0 < k/Q < 1/2$ for simplicity, fermionic oscillators with the smallest energy are $\overline{\psi}_{A,-1/2+k/Q}$ in the left moving part and $\widetilde{\psi}^{A}_{-1/2+k/Q}$ in the right moving part. By exciting the ground state with these oscillators, we have four massless states for each k

$$\overline{\psi}_{A,-1/2+k/Q}|0\rangle_L \otimes \widetilde{\psi}^{B}_{-1/2+k/Q}|0\rangle_R\,. \tag{3.14}$$

Therefore, there is no tachyonic mode. These massless modes correspond to blow up moduli parameters of the singularity.

Type 0 on $\mathbb{R}^4/\mathbb{Z}_Q$. In the type-0 theory, in addition to the states obtained in type-II case, states with opposite fermion numbers are allowed. Because we are focusing on tachyonic states now, the only oscillators we can use are $z^{A}_{-k/Q}$ in the left moving part and $\overline{\widetilde{z}}_{A,-k/Q}$ in the right moving part. (We assume $0 < k/Q < 1/2$.) By exciting both vacuums in the left and right moving part by P oscillators, we obtain

$$z^{A_1}_{-k/Q}\cdots z^{A_P}_{-k/Q}|0\rangle_L \otimes \overline{\widetilde{z}}_{B_1,-k/Q}\cdots\overline{\widetilde{z}}_{B_P,-k/Q}|0\rangle_R\,. \tag{3.15}$$

These states belong to

$$(\mathbf{P}+\mathbf{1})\times(\mathbf{P}+\mathbf{1}) = (\mathbf{2P}+\mathbf{1})+(\mathbf{2P}-\mathbf{1})+\cdots+\mathbf{1}\,, \tag{3.16}$$

representation of $\mathrm{SU}(2)_R = \mathrm{SU}(2)_D$ symmetry. The mass of these states is

$$M^2 = M_T^2 + \frac{4}{l_s^2}\frac{k(P+1)}{Q}\,, \tag{3.17}$$

where $M_T^2 = -2/l_s^2$ is mass^2 of the bulk tachyon field. For small P (3.15) gives tachyonic modes.

Type II on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_S)}$. In type-II theory on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_S)}$, we have the following spectrum. For even k we have the same spectrum with type II on $\mathbb{R}^4/\mathbb{Z}_{2Q}$ and we have no tachyonic modes. For odd k, we should take account of the difference between $\mathbb{Z}_{2Q}$ and $\mathbb{Z}_{2Q}^{(F_S)}$. In GS formalism, the boundary condition for the fermion fields S^α is given as

$$S^\alpha(\sigma+2\pi) = (-)^k\,\mathrm{U}(\gamma^k)S^\alpha(\sigma)\,, \tag{3.18}$$

where $\mathrm{U}(\gamma^k)$ is a spinor representation of $\gamma^k \in \mathrm{SU}(2)_L$. The nontrivial monodromy causes the negative sign for odd k. This change of the boundary condition for the GS fermions corresponds to the reversal of the GSO projection in the NSR formalism. Due to this change of the GSO projection, we have the same tachyonic spectrum with type 0 on $\mathbb{R}^4/\mathbb{Z}_{2Q}$. Therefore, the tachyonic spectrum of this theory is obtained by replacing Q by $2Q$, and k by $2k'$,where $k' \in \mathbb{Z}+1/2$ in (3.15) and (3.17) as follows:

$$z^{A_1}_{-k'/Q}\cdots z^{A_P}_{-k'/Q}|0\rangle_L \otimes \overline{\widetilde{z}}_{B_1,-k'/Q}\cdots\overline{\widetilde{z}}_{B_P,-k'/Q}|0\rangle_R\,. \tag{3.19}$$

$$M^2 = M_T^2 + \frac{4}{l_s^2}\frac{k'(P+1)}{Q}\,, \tag{3.20}$$

The multiplicity of each level is given by (3.16).

Type 0 on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_R)}$. On this orbifold, the boundary condition for the right moving NSR fermion fields $\widetilde{\psi}^A$ is given as

$$\widetilde{\psi}^A(\sigma+2\pi) = -(-)^k e^{2\pi ik/2Q}\widetilde{\psi}^A(\sigma). \tag{3.21}$$

Because of the extra factor $(-)^k$ we have no NS-NS sector for odd k. For even k, we have the same spectrum with type 0 on $\mathbb{R}^4/\mathbb{Z}_{2Q}$ before $\mathbb{Z}_{2Q}^{(F_R)}$ projection. However, these tachyonic states have $(-1)^{F_R} = -1$ and are projected out by $\mathbb{Z}_{2Q}^{(F_R)}$ projection. Therefore, this configuration has no tachyonic spectrum.

4. Gauge fields on NS5-branes

In the RR sector, the zero point energy contributions from bosons and fermions always cancel and ground states always give massless modes. Therefore, tachyonic modes do not exist. However, it is worth seeing how these twisted modes reproduce gauge fields on coincident Q NS5-branes.

By the argument of S-duality, it is presumed that the gauge groups on Q coincident NS5-branes are $\mathrm{U}(Q)$ in type-IIB theory and $\mathrm{U}(Q)^2$ in type-0B theory. In the type A case, although we cannot define 'gauge groups', it is expected that Q^2 self-dual tensor multiplets and Q^2 pairs of self-dual and anti-self-dual tensor multiplets appear on type-IIA and type-0A NS5-branes respectively. The Cartan part of these gauge fields are reproduced as massless twisted modes of the RR sector and zero modes of the RR fields in the bulk as follows.

By using usual orbifold procedure with paying attention to the effect of the operators F_s and F_R, we obtain the massless spectrum displayed in table 2 in each case.

The spectra in this table contain the contribution of zero modes of bulk RR fields. The zero modes always belong to the same representation with twisted modes for even k and it is convenient to regard them as modes of $k = 0$.

5. Tachyon modes on NS5-brane solution

In section 3, we showed that tachyonic twisted modes arose in type 0 on $\mathbb{R}^4/\mathbb{Z}_Q$ and type II on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_S)}$, both which are dual to type-0 NS5-branes. What are counterparts for these tachyonic modes on the NS5-branes side? We show that these can be reproduced as modes of the bulk tachyon fields localized on a NS5-brane classical solution.

In order to analyze modes, we need a classical solution of supergravity. The relevant part of supergravity action is

$$S = \frac{1}{(2\pi)^7 l_s^8}\int d^{10}x\sqrt{-g}\frac{1}{e^{2\phi}}\left(R + 4(\partial\phi)^2 - \frac{(2\pi l_s)^4}{2\cdot 3!(2\pi)^2}H_3^2\right). \tag{5.1}$$

orbifold	rep. for each k	fields
type IIA on $\mathbb{R}^4/\mathbb{Z}_Q$ (dual to Q IIB NS5)	$\mathbf{2_R} \times \mathbf{2_L}$	Q vectors
type IIB on $\mathbb{R}^4/\mathbb{Z}_Q$ (dual to Q IIA NS5)	$\mathbf{2_L} \times \mathbf{2_L}$	Q self-dual 2-forms and Q scalars
type 0A on $\mathbb{R}^4/\mathbb{Z}_Q$ (dual to Q 0B NS5)	$\mathbf{2_R} \times \mathbf{2_L}$ $+\mathbf{2_L} \times \mathbf{2_R}$	$2Q$ vectors
type 0B on $\mathbb{R}^4/\mathbb{Z}_Q$ (dual to Q 0A NS5)	$\mathbf{2_L} \times \mathbf{2_L}$ $+\mathbf{2_R} \times \mathbf{2_R}$	Q 2-forms and $2Q$ scalars
type IIA on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_s)}$ (dual to Q 0A NS5)	$\mathbf{2_L} \times \mathbf{2_R}$ (for even k) $\mathbf{2_R} \times \mathbf{2_L}$ (for odd k)	$2Q$ vectors
type IIB on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_s)}$ (dual to Q 0A NS5)	$\mathbf{2_L} \times \mathbf{2_L}$ (for even k) $\mathbf{2_R} \times \mathbf{2_R}$ (for odd k)	Q 2-forms and $2Q$ scalars
type 0A on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_R)}$ (dual to Q 0A NS5)	$\mathbf{2_R} \times \mathbf{2_L}$ (for even k) no RR sector (for odd k)	Q vectors
type 0B on $\mathbb{R}^4/\mathbb{Z}_{2Q}^{(F_R)}$ (dual to Q 0A NS5)	$\mathbf{2_L} \times \mathbf{2_L}$ (for even k) no RR sector (for odd k)	Q self-dual 2-forms and Q scalars

Table 2: The massless twisted modes in the RR sector and zero modes of the RR fields in the bulk. 'rep. for each k' represents representations of little group $\mathrm{SO}(4) \subset P(1,5)$ on the NS5-branes. The modes with $k = 0$ represent the zero modes of the RR fields.

This is common to type-IIA, IIB, 0A and 0B supergravities. We use the convention in which the gauge flux is quantized as follows.

$$\oint_{S^3} H_3 = 2\pi Q\,. \tag{5.2}$$

The classical solution representing parallel NS5-branes is [14, 15]

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + f(x^i)(dx^i)^2\,, \qquad e^{2\phi} = f(x^i)\,. \tag{5.3}$$

where x^μ ($\mu = 0, \ldots, 5$) and x^i ($i = 6, \ldots, 9$) are coordinates along parallel and transverse directions, respectively. The function $f(x^i)$ is a harmonic function on the x^i plane, which is related to a fivebrane density $\rho_{\mathrm{NS5}}(x^i)$ by the following Laplace equation:

$$\sum_{i=6}^{9} \left(\frac{\partial}{\partial x^i}\right)^2 f(x^i) = -(2\pi l_s)^2 \rho_{\mathrm{NS5}}(x^i)\,. \tag{5.4}$$

A solution for Q coincident NS5-branes located at a point $x^i = 0$ is obtained by setting

$$\rho_{\mathrm{NS5}}(x^i) = Q \sum_{n=-\infty}^{\infty} \delta\left(x^6, x^7, x^8, x^9 - 2\pi n R\right), \tag{5.5}$$

where we took the compactification of the x^9 direction into account. In this case, $f(x^i)$ is given as

$$f(x^i) = 1 + \sum_{n=-\infty}^{\infty} \frac{r_0^2}{(x^i)^2 + (x^9 - 2\pi n R)^2}\,, \qquad r_0^2 = l_s^2 Q\,. \tag{5.6}$$

Tachyonic modes obtained in section 3 belong to $(\mathbf{P}+\mathbf{1})\times(\mathbf{P}+\mathbf{1})$ representation of $\mathrm{SU}(2)_D = \mathrm{SU}(2)_L$ rotation symmetry. If this is $(\mathbf{P}+\mathbf{1}, \mathbf{P}+\mathbf{1})$ representation of $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$, the structure of the spectrum is similar to that of modes of a scalar field in $\mathrm{SO}(4) = \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ symmetric potential. This seems to imply that these modes are localized in the near horizon region $|x^i| \ll R$ where the effect of compactification can be neglected. However, as we will show below, this is not true. This degeneracy of spectrum is accidental one rather than result of some symmetry.

At first, let us consider the region where $r = (\sum_{i=6}^{9} x_i^2)^{1/2}$ satisfies

$$r \ll r_0 \text{ and } r \ll R\,. \tag{5.7}$$

In this region, the metric reduce to

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + r_0^2 \frac{dr^2}{r^2} + r_0^2 d\Omega_3^2\,. \tag{5.8}$$

This metric has structure $\mathbb{R}^6 \times \mathbb{R}^+ \times S^3$. Unfortunately, the infinitely long throat structure of this solution implies tachyon modes cannot be quantized. Actually, using the action of tachyon field

$$S = -\frac{1}{(2\pi)^7 l_s^8} \int d^{10}x \sqrt{-g} \frac{1}{2e^{2\phi}} \left[(\partial T)^2 + M_T^2 T^2 \right], \tag{5.9}$$

we obtain the following equation of motion:

$$\Delta_6 T = \left[M_T^2 - \frac{1}{r_0^2 r} \partial_r r^3 \partial_r - \frac{1}{r_0^2} \Delta_{S^3} \right] T\,, \tag{5.10}$$

where Δ_S^3 is laplacean on unit S^3 and Δ_6 is laplacean along x^μ, whose eigenvalue gives the mass^2 of the modes on the fivebrane. The eigenfunction of the operator in the second term in the bracket in the right hand side is r^n and its eigenvalue is $s = n(n+2)/r_0^2$. If $s < 1/r_0^2$, the wave function diverges at $r \to 0$ or $r \to \infty$. They represent modes falling into horizon and modes living outside the throat. If $s > 1/r_0^2$, the wave function can propagate along the throat. The existence of these mode are connected with the non-vanishing of Hawking radiation and that of absorption cross section for bulk field [3, 5]. This result implies that tachyon modes cannot be trapped in the near horizon region. So, we need to consider modes outside it. If $r_0 \ll R$, the outside of the region (5.7) is flat and modes cannot be trapped. Therefore we assume

$$R \ll r_0\,. \tag{5.11}$$

In this case, the region with $r \gg R$ is described by a smeared NS5-brane solution. A smeared NS5-brane solution with charge Q distributed around the circle uniformly is specified by the following NS5-brane density:

$$\rho_{\mathrm{NS5}}(x^i) = \frac{Q}{2\pi R}\delta^3(x^6, x^7, x^8)\,. \tag{5.12}$$

The harmonic function $f(x^i)$ is given by

$$f(x^i) = 1 + \frac{r_0'}{r}\,, \qquad r_0' = \frac{l_s^2 Q}{2R}\,. \tag{5.13}$$

Now, r is defined by $r = \sum_{i=6,7,8}(x^i)^2$. The near horizon ($r \ll r_0'$) metric of this solution is

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + \frac{r_0'}{r}(dz^2 + dr^2) + r_0' r d\Omega_2^2\,. \tag{5.14}$$

We should notice that (5.11) is equivalent to $r_0 \ll r_0'$ and the region described by (5.14) always exists.

On this manifold, we have the following equation of motion of tachyon.

$$\Delta_6 T = \left[M_T^2 - \frac{r}{r_0'}\partial_9^2 - \frac{1}{r_0' r}\partial_r r^2 \partial_r - \frac{1}{r_0' r}\Delta_{S^2}\right] T\,. \tag{5.15}$$

This differential equation has the same structure with the Shrödinger equation for a particle in a Coulomb potential. Let us decompose T as

$$T = \phi(r) Y_{L,m}(\theta_1, \theta_2) e^{-ip_9 x^9} e^{-ip_\mu x^\mu}\,, \tag{5.16}$$

where $Y_{L,m}(\theta_1,\theta_2)$ is the spherical harmonic function on S^2 and p_9 and p_μ are momenta along x^9 and x^μ, respectively. The mass on the fivebrane M is given by $M^2 = -\eta^{\mu\nu}p_\mu p_\nu$. Then, the equation reduced to

$$M^2\phi = \left[M_T^2 + \frac{r}{r_0'}p_9^2 - \frac{1}{r_0' r}\partial_r r^2 \partial_r + \frac{1}{r_0' r}L(L+1)\right]\phi\,. \tag{5.17}$$

The Kaluza-Klein momenta p_9 is quantized as

$$p_9 = \frac{m}{R}\,, \tag{5.18}$$

where m is an integer in the case of S^1 compactification and is a half odd integer in the case of $S^1_{(F_R)}$ compactification because the tachyon field has $(-1)^{F_R} = -1$. Via T-duality, m is transformed into wrapping number, and is identified with $k \in \mathbb{Z}$ in (3.15) and (3.17) or $k' \in \mathbb{Z} + 1/2$ in (3.19) and (3.20). One might be afraid that (5.17) cannot reproduce the twisted mode spectrum because the Kaluza-Klein momentum depends on the compactification radius R while the dependence is absent on the twisted mode side. This, however, is not the case. By rescaling of radial coordinate

$$\rho = 2p_9 r, \tag{5.19}$$

we obtain the R-independent expression

$$\frac{1}{\rho^2}\partial_\rho \rho^2 \partial_\rho \phi = \left[\frac{1}{4} - \frac{\lambda}{\rho} + \frac{L(L+1)}{\rho^2}\right]\phi\,, \tag{5.20}$$

where we defined λ as follows:

$$\lambda = \frac{r_0'}{2p_9}(M^2 - M_T^2)\,. \tag{5.21}$$

This number corresponds to the principal quantum number of a particle in the Coulomb potential and is quantized as we show below.

For the tachyon field not to diverge at the origin and infinity, the function $\phi(\rho)$ should behave like the following way asymptotically:

$$\phi(\rho) \sim \rho^L \quad (\rho \to 0)\,, \qquad \phi(\rho) \sim e^{-\rho/2} \qquad (\rho \to \infty)\,. \tag{5.22}$$

Let us expand the function $\phi(\rho)$ as follows:

$$\phi(\rho) = \sum_n a_n \rho^n e^{-\rho/2}\,. \tag{5.23}$$

Then, the differential equation (5.20) gives the following relation among the coefficients a_n:

$$[n(n+1) - L(L+1)]\, a_n = (n-\lambda) a_{n-1}\,. \tag{5.24}$$

If we put $n = L$, this equation gives $a_{L-1} = 0$ and it is consistent with the $\rho \to 0$ behavior of $\phi(\rho)$. To reproduce $r \to \infty$ behavior in (5.22), only finite number of coefficients a_n can be non-zero. For this condition to be satisfied, the following values are allowed for λ:

$$\lambda = L+1, L+2, L+3, \ldots \tag{5.25}$$

As a result, we obtain the mass of the mode.

$$M^2 = M_T^2 + \frac{4m}{l_s^2 Q}\lambda\,. \tag{5.26}$$

If we identify λ with $P+1$, this result exactly reproduces the tachyonic spectrum of the twisted modes (3.17) and (3.20)!

For fixed $\lambda(=P+1)$, all states with L smaller than λ are degenerate. Therefore, multiplicity is

$$(\mathbf{2P+1}) + (\mathbf{2P-1}) + \cdots + \mathbf{1} = (\mathbf{P+1}) \times (\mathbf{P+1})\,. \tag{5.27}$$

This is completely same with the multiplicity (3.16) of the twisted modes.

JHEP01(2000)018

6. Discussion

In this paper, we restricted our attention to the tachyonic modes by the following reason.

- The structure of spectrum of twisted modes is very simple. Because many oscillators contribute to massive states, the massive spectrum is more complicated.
- On classical solution side, the identification of modes is very easy. Tachyonic modes can come from only the tachyon field. In order to identify massive twisted modes with modes of bulk fields, we need to analyze their quantum numbers in more detail.
- Because the tachyon field is a scalar, it is very easy to calculate the eigen mode on the classical solution.

Of course, it is an interesting problem to investigate to what extent this correspondence holds. Especially, it is important to understand how the gauge fields on NS5-branes are described in the context of supergravity. However, leaving technical obstacles, it seems difficult to reproduce all gauge fields obtained as twisted modes. According to the argument in the AdS/CFT correspondence, supergravity modes corresponds only to gauge invariant operator. In fact, in [6], only the gauge singlet part of gauge fields are obtained as modes of bulk RR fields. Therefore, classical modes on NS5-brane solutions may be matched off against gauge singlet twisted modes at orbifold singularity.

In the derivation of the tachyonic spectrum, we used two approximations. One is neglect of the first term in (5.13). This is same with taking the limit $R \to 0$ on the NS5-brane side.[1] The other is neglect of the localization of the NS5-branes. In the rest of this section, let us discuss the conditions for these approximation are applicable.

In section 5, we used the smeared NS5-brane metric (5.14) to obtain the discrete tachyonic modes. The metric (5.14) is available only in the region

$$R \ll r \ll r_0' \,. \tag{6.1}$$

Therefore, analysis of tachyon modes is valid only when the support of the wave function of the mode is in this region. Using (5.19), the upper bound in (6.1) is rewritten as

$$\rho \ll \frac{l_s^2 Q m}{R^2} \,. \tag{6.2}$$

[1] This does *not* mean that the tachyonic modes get large Kaluza-Klein momentum and become massive modes because R represents the compactification radious of asymptotic region, and near NS5-branes the proper radious is represented by the metric (5.3). Even in the $R = 0$ case, the background geometry, except infinity point, is not singular and the tachyonic modes still exist.

It is known that the following equation holds for a solution of (5.20)

$$\overline{\rho^{-1}} \equiv \frac{\int_0^\infty (1/\rho)\rho^2\phi^2(\rho)d\rho}{\int_0^\infty \rho^2\phi^2(\rho)d\rho} = \frac{1}{2\lambda}\,. \tag{6.3}$$

In the context of the quantum mechanics on the Coulomb potential, this implies that expectation value of the potential energy is proportional to the total energy. So, we can use $P \sim \lambda$ as a typical value of ρ. Then we obtain the following bound:

$$P \ll \frac{l_s^2 Qm}{R^2}\,. \tag{6.4}$$

On the other hand, on the Kaluza-Klein monopole side, we neglect the first term in (2.3) to obtain the orbifold metric (2.4). This is possible for $r^2 \sim \widetilde{R}Q|x^i| \ll \widetilde{R}^2Q^2$. The value $r^2 = |z^A|^2$ can be estimated as an expectation value of the following operator:

$$\begin{aligned}\frac{1}{2\pi}\int_0^{2\pi} &: z^A(\sigma)\overline{z}_A(\sigma) : d\sigma = \\ &= l_s^2 \sum_{m=-\infty}^{\infty}\sum_{A=1,2}\left(\frac{: z^A_{m-k/N}\overline{z}_{A,-m+k/N} :}{(m-k/N)^2} + \frac{: \widetilde{z}^A_{m+k/N}\overline{\widetilde{z}}_{A,-m-k/N} :}{(m+k/N)^2}\right).\end{aligned} \tag{6.5}$$

On the states (3.15), this gives

$$r^2 \sim l_s^2\frac{QP}{k}\,. \tag{6.6}$$

Therefore, the applicable limit is

$$\frac{P}{Qk} \ll \frac{l_s^2}{R^2}\,. \tag{6.7}$$

This is the same as (6.4).

Finally, let us discuss the lower bound in (6.1). This condition demand the tachyon modes not to see the localization of NS5-branes. If we use P as the typical value of ρ again, the lower bound in (6.1) is rewritten as

$$\frac{k}{P} \ll 1\,. \tag{6.8}$$

In [16], the relation between a Kaluza-Klein monopole and a localized NS5-brane are argued and it is shown that zero modes of NS-NS two-form field on the Kaluza-Klein monopoles, which are also regarded as twisted modes at the singularity, play an important role. However, in our analysis, we did not take account of the interactions among twisted sectors. Therefore, it is valid only when the interaction can be neglected. This argument seems consistent with condition (6.8) because, roughly speaking, the interaction between the B field and strings is proportional to the winding number k and the probability that a string exist near the singularity becomes smaller when P becomes larger.

Acknowledgments

I would like to thank B. Zwiebach for careful reading of the manuscript. I also appreciate hospitality of the organizers of Summer Institute '99 at Yamanashi, Japan, where a part of this work was discussed.

References

[1] N. Seiberg, *New theories in six dimensions and matrix description of M-theory on T^5 and T^5/Z_2*, *Phys. Lett.* **B 408** (1997) 98 [hep-th/9705221].

[2] N. Seiberg and S. Sethi, *Comments on Neveu-Schwarz five-branes*, *Adv. Theor. Math. Phys.* **1** (1998) 259 [hep-th/9708085].

[3] J.M. Maldacena and A. Strominger, *Semiclassical decay of near-extremal fivebranes*, *J. High Energy Phys.* **12** (1997) 008 [hep-th/9710014].

[4] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS5-branes and holography*, *J. High Energy Phys.* **10** (1998) 004 [hep-th/9808149].

[5] S. Minwalla and N. Seiberg, *Comments on the IIA NS5-brane*, *J. High Energy Phys.* **06** (1999) 007 [hep-th/9904142].

[6] C.G. Callan Jr., J.A. Harvey and A. Strominger, *Worldbrane actions for string solitons*, *Nucl. Phys.* **B 367** (1991) 60.

[7] H. Ooguri and C. Vafa, *Two-dimensional black hole and singularities of CY manifolds*, *Nucl. Phys.* **B 463** (1996) 55 [hep-th/9511164].

[8] B. Craps and F. Roose, *Ns fivebranes in type-0 string theory*, *J. High Energy Phys.* **10** (1999) 007 [hep-th/9906179].

[9] Y. Imamura, *Branes in type-0/type-II duality*, hep-th/9906090.

[10] O. Bergman and M.R. Gaberdiel, *Dualities of type-0 strings*, *J. High Energy Phys.* **07** (1999) 022 [hep-th/9906055].

[11] S.W. Hawking, *Gravitational instantons*, *Phys. Lett.* **A 60** (1977) 81.

[12] G.W. Gibbons and S.W. Hawking, *Classification of gravitational instanton symmetries*, *Comm. Math. Phys.* **66** (1979) 291.

[13] E. Bergshoeff, C. Hull and T. Ortin, *Duality in the type-II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081].

[14] A. Strominger, *Heterotic solitons*, *Nucl. Phys.* **B 343** (1990) 167.

[15] M.J. Duff and J.X. Lu, *Elementary five-brane solutions of $D = 10$ supergravity*, *Nucl. Phys.* **B 354** (1991) 141.

[16] R. Gregory, J.A. Harvey and G. Moore, *Unwinding strings and T-duality of Kaluza-Klein and H-monopoles*, *Adv. Theor. Math. Phys.* **1** (1997) 283 [hep-th/9708086].